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ASTEROSEISMOLOGY - THEORY AND PHENOMENOLOGY / /0

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I. Introduction

Asteroseismology is commonly understood to mean the study of normal-mode pulsations in stars that, like the Sun, display a large number of simultaneously excited modes. The idea of learning about a physical system by examining its oscillation modes is of course an old one in physics, but it is only fairly recently that data of sufficient quality have become available to apply this technique to stars.

The Sun is (and will likely remain) the outstanding example of the progress that can be made using seismological methods. Seismic studies of the Sun have succeeded in mapping the variation of sound speed with depth in the Sun, and the variation of angular velocity with both depth and latitude; they have measured the depth of the Sun's convective envelope, and they have begun to be used to estimate the helium abundance in the convection zone and to reveal at least some of the subsurface structure of solar activity.

Many stars besides the Sun may be also be amenable to asteroseismic analysis. Stars of roughly solar type should of course behave in ways similar to the Sun, and stars of this sort form a large fraction of the potential targets for asteroseismology. But several other kinds of star (δ Scuti stars, roAp stars, and the pulsating white dwarfs) also have the desired pulsation characteristics. Pulsations in some of these stars are, for various reasons, much easier to observe than in the Sun-like stars; indeed, to date virtually all unambiguous observations of multi-mode pulsators relate to these other categories of stars.

Regardless of the type of star or the mechanism driving its pulsations, we will not in the foreseeable future have as much pulsation information about other stars as we have about the Sun. A very large majority of the 10⁷ modes seen in the Sun have horizontal wavelengths that are a small fraction of a solar radius. When averaged over the solar disk (as a distant observer would do), the perturbations due to these modes average to zero, rendering them undetectable. It is only in special circumstances that modes with angular degree greater than about 3 are observable on distant stars; the number of modes that may be observed is therefore likely to be at most a few tens. Nevertheless, since oscillation mode frequencies are arguably the most precise measurements relating to a star that we can make, a few tens of such frequencies may still be of great importance to our understanding of stellar structure and evolution.

In what follows I shall try to indicate what sorts of astrophysical questions one might answer if high-quality oscillations data (such as might be obtained with FRESIP) were available. To this end, I shall first review some of the necessary background physics and jargon. I shall then discuss two kinds of stars: oscillating white dwarfs (chosen because they presently are the best example of what seismological methods can do, and are doing), and stars like the Sun (chosen because their oscillation properties have been well studied theoretically, and because they are observationally intractable from the ground, but are within reach of FRESIP). Last, I shall give a brief listing of the salient observational

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characteristics of pulsating stars that might reasonably be observed with FRESIP. Most of the following discussion is taken (in rearranged and abridged form) from a review of asteroseismology prepared for Annual Reviews of Astronomy and Astrophysics (Brown & Gilliland 1994). For a more complete treatment of these topics, and for a much more complete reference list, please see that paper.

II. Terminology and Basic Physics

Stars that display pulsations may be described with reasonable accuracy as spheres. For this reason it is possible and convenient to write the pulsation eigenmodes as the product of a function of radius and a spherical harmonic. The spatial and temporal variation of a perturbation to the star's mean state are then

$$\xi_{nlm}(r,\theta,\phi,t) = \xi_{nl}(r) Y_l^m(\theta,\phi) e^{-i\omega_{nlm}t}.$$
 (1)

Here ξ is any scalar perturbation associated with the mode (e.g., the radial displacement); r, θ, ϕ, t are the radial coordinate, the colatitude, the longitude, and time, respectively. The mode's radial order n is usually identified with the number of nodes in the eigenfunction that exist between the center of the star and its surface. Since it deals with the depth structure, n is not accessible to direct observation. The angular degree l is the product of the stellar radius R_* and the total horizontal wavenumber of the mode; modes with large values of l display many sign changes across a stellar hemisphere, and hence are usually unobservable on distant stars. The azimuthal order m is the projection of l onto the star's equator; it is therefore restricted to be less than or equal to l in absolute value. Note that p-modes may be purely radial (l = 0), but that g-modes, since they are driven by buoyancy forces, must involve a variation in the horizontal coordinates, and hence always have $l \geq 1$. The mode frequency ω_{nlm} generally depends on n and l in complicated ways, depending on the restoring forces responsible for the pulsation and on the structure of the star. In particular, there is generally no simple harmonic relation between the frequencies of modes with (for instance) given l and successive values of n. For stars that are truly spherically symmetric, mode frequencies depend only upon n and l, and are independent of m. Any condition that breaks the spherical symmetry (such as rotation about an axis, or the presence of magnetic fields) can lift this frequency degeneracy.

Observations of stellar pulsations usually involve either the photometric intensity or the radial velocity. The displacements of the stellar plasma cause Doppler shifts directly; the accompanying compressions or displacements from equilibrium height also cause temperature changes, resulting in perturbations to the observed intensity. The oscillation parameters that one may observe on stars include not only the mode frequencies, but also amplitudes and linewidths. Frequencies usually carry the most information, because they can be measured accurately and because they can usually be calculated with good accuracy considering only adiabatic effects. Because of the relatively poor current understanding of the physics of mode driving and damping, henceforth I shall concentrate mostly on the information that may be inferred from mode frequencies.

Although mode frequencies depend in complicated ways on the stellar structure, there is a useful limit (that in which $n \gg l$) in which simple asymptotic formulae give useful approximations to the true frequency behavior (Tassoul 1980). For p-modes, one finds

$$\nu_{nl} \cong \Delta\nu_0 \left(n + \frac{l}{2} + \epsilon \right) - \frac{A L^2 - \eta}{\left(n + l/2 + \epsilon \right)}, \tag{2}$$

where $\Delta\nu_0$, A, ϵ , and η are parameters that depend on the structure of the star, and $L^2 \equiv l(l+1)$. If the parameters A and η were zero, one would therefore find p-mode frequencies to fall in a regular picket fence pattern with frequency spacing $\Delta\nu_0/2$: modes

with odd l would fall exactly halfway between modes with even l, and modes with different n at a given l would always be separated in frequency by multiples of $\Delta \nu_0$. The parameter $\Delta \nu_0$, termed the large separation, is simply related to the sound travel time through the center of the star:

$$\Delta\nu_0 = \left(2\int_0^{R_*} \frac{dr}{c}\right)^{-1},\tag{3}$$

where c is the local sound speed and R_* is the stellar radius. Consideration of the virial theorem (Cox 1980) shows that this travel time is related to the mean density of the star, so that

$$\Delta\nu_0 \cong 135 \left(\frac{M_*}{R_*^3}\right)^{1/2} \mu \text{Hz}, \tag{4}$$

where M_{\star} and R_{\star} are the stellar mass and radius in solar units. Eq. (4) holds exactly for homologous families of stars, but it is obeyed quite closely even for stars that are not homologous, such as stars of different mass along the main sequence (Ulrich 1986). The large separation is thus easily interpreted in terms of the stellar structure, and moreover it is likely to be straightforward to observe, even in noisy stellar oscillation data.

Parameters A and parameters ϵ and η in Eq. (2) have to do with the structure near the center of the star and near the surface, respectively. Modes with different degree l penetrate to different depths within the star. Modes with l=0 have substantial amplitude even at the center: those with higher values of l avoid a region in the stellar core that grows in radius as l increases. This difference in the region sampled by modes with different l leads to the second term on the right-hand side of Eq. (2), removing the frequency degeneracy between modes that differ by (say) -1 in n and +2 in l. This effect is often parameterized in terms of the small separation, defined as $\delta_{nl} \equiv \nu_{n+1,l} - \nu_{n,l+2}$. The small separation may be written as an integral analogous to that in Eq. (3) (Däppen et al. 1988):

$$\delta_{n,l} = \Delta \nu_0 \frac{(l+1)}{2\pi^2 \nu_{nl}} \int_0^{R*} \frac{dc}{dr} \frac{dr}{r} \,. \tag{5}$$

The small separation is thus sensitive to sound speed gradients, particularly in the stellar core. Since these gradients change as nuclear burning changes the molecular weight distribution in the star's energy-producing region, the small splitting contains information about the star's evolutionary state.

In the asymptotic limit $n \gg l$, g-mode periods (not frequencies) become almost equally spaced:

$$T_{nl} = \frac{T_0[n + l/2 + \delta]}{L} , \qquad (6)$$

where T_{nl} is the period of a g-mode, T_0 and δ are parameters that play roles similar to those of $\Delta\nu_0$ and ϵ in Eq. (2), and the other symbols have their previous meanings. The asymptotic period T_0 depends upon an integral of the Brunt-Väisälä frequency N throughout the star (Tassoul 1980):

$$T_0 = 2\pi^2 \left[\int_0^{R_*} \frac{N}{r} dr \right]^{-1} . (7)$$

Another important issue addressable with pulsation data is the frequency splitting of modes with l > 0 by the solar rotation (see, e.g., Hansen et al. 1977). In the simple case

in which angular velocity is independent of latitude, the frequencies of modes within a multiplet are given by

$$\nu_{nlm} = \nu_{nl0} + \frac{m\beta_{nl}}{2\pi} \int_0^{R_*} \Omega(r) K_{nl}(r) dr , \qquad (8)$$

where β_{nl} is a correction factor of order unity accounting for Coriolis forces, $\Omega(r)$ is the solar angular velocity, and K_{nl} is a unimodular kernel that is roughly proportional to the local energy density in the mode. Thus, the splitting of low-l multiplets (sets of modes with the same n and l but different m) depends somewhat on the angular velocity near the Sun's center, where only eigenmodes with small l have substantial amplitude.

III. Pulsating White Dwarfs

The utility of asteroseismology is perhaps best seen in its applications to white dwarfs. Here, one need not talk about potential rewards or possible conclusions; because of a fortunate confluence of observational and theoretical progress, meaningful results about white dwarfs are available now, and more are on the way. Oscillations in white dwarfs are seen as photometric fluctuations with amplitudes that range from roughly 0.3 magnitude down to the limits of detectability; typical periods fall in the range between 100 s and 2000 s. The combination of fairly large amplitudes, short periods, and many excited modes makes the variable WD stars ideal subjects for asteroseismology. Indeed, the successes of WD pulsation studies provide the best current example of asteroseismological methods, and many of the techniques used by the WD community may be viewed as archetypes for the investigation of other kinds of stars.

Strong surface gravity on WD stars discriminates against p-mode pulsations (which involve vertical motions), and favors g-modes (in which the motions are predominantly horizontal). But g-modes may propagate only in regions with stable density stratifications. In white dwarfs, g-modes are therefore confined to the stellar envelope, above the degenerate core. The eigenfunctions may be trapped in an even stronger sense, however, by the rapid jumps in fluid properties that occur at boundaries between different composition layers (Kawaler & Weiss 1990). Although Eq. (6) continues to describe the mean period spacing, the period separation between modes with consecutive values of n shows variations near trapped modes. The detailed variation of period spacing with mode period leads to powerful diagnostics of the near-surface structure.

One of the most straightforward and yet informative applications of WD seismology has been the direct measurement of WD evolutionary timescales, using observations of period changes in WD pulsators. Some pulsating white dwarfs have power spectra that are dominated by one or a few pulsation modes, and in many such cases the frequencies of these modes have proved to be extremely stable. Changes in g-mode periods in evolving white dwarfs may be thought of as resulting from a combination of changes in the temperature and changes in the radius:

$$\frac{\dot{\Pi}}{\Pi} = -a\frac{\dot{T}}{T} + b\frac{\dot{R}}{R} \,, \tag{9}$$

where Π is the period, T and R are the temperature and radius, and a and b are positive constants of order unity (Winget et al. 1983). The best characterized case of period change is the prototypical DOV star PG1159-035. As illustrated in Figure 1, this star oscillates in many modes (125 at last count, Winget et al. 1991), but a few of these have substantially larger amplitudes than the others. One of these, with a period near 516 s, has been found to have nearly constant amplitude and consistent variation of phase with time over the interval 1979-1989. The rate of period change is quite well determined in this case: $\dot{\Pi} = (-2.49 \pm 0.06) \times 10^{-11}$, corresponding to an evolutionary timescale of

about 0.7×10^6 y, with the period decreasing with time. The observed timescale is about what stellar models predict. Unfortunately, however, evolutionary models of DOV stars show that the periods of typical g-modes should *increase* with time, as cooling dominates the effects of global contraction in Eq. (9) (Kawaler et al. 1986). The explanation for this contradiction is probably that the 516 s mode is not typical; it is a trapped mode, for which changes in the stratification dominate effects of changing overall structure. Kawaler (1993) finds that detailed models of PG1159-035 give trapped modes with very nearly the observed period; these modes display negative values of Π , although no model gives negative period changes that are as large in magnitude as that observed.

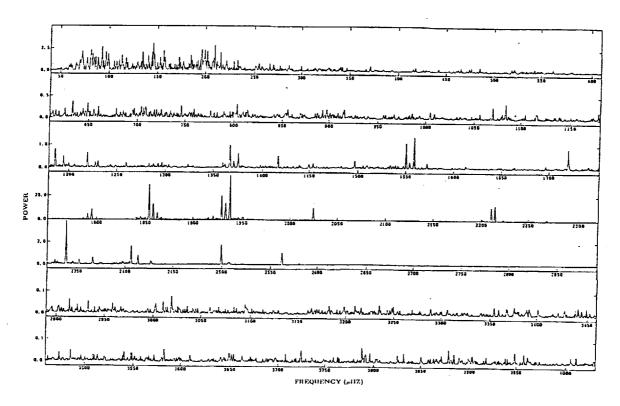


Fig. 1. Power spectrum of the intensity pulsations of PG1159-035, as observed by the Whole Earth Telescope. Note the vertical scale differs from panel to panel, to accommodate the large dynamic range. From Winget et al. (1991).

Interpretation of the power spectrum shown in Figure 1 has yielded a tremendous amount of information concerning PG1159-035 (see Winget et al. 1991, and references therein). Among the most firmly established and interesting results are the following: (1) Virtually all of the peaks seen in the power spectrum may be identified as members either of triplets (with l=1) or quintuplets (with l=2). Conclusive evidence for the l identification is provided by the ratio of period spacings for the two kinds of multiplets. Models show that this ratio should be within a few percent of its asymptotic value of $\sqrt{3}\cong 1.73$, while the observed ratio is 1.72. (2) The source of the frequency splitting within multiplets is apparently rotation of the star. This identification is consistent with the presence of 2l+1 components in each multiplet (as in Eq. 8), and moreover the observed frequency splittings for triplets and quintuplets are consistent with the calculated values of β_{nl} from Eq. (8). The inferred rotation period for the star (assuming uniform rotation) is 1.38 ± 0.01 days. (3) The presence of a significant global magnetic field would induce a

component of the frequency splitting proportional to m^2 . No such symmetric component is observed; the observed limit on its magnitude implies a maximum global field strength of about 6000 G. (4) The mean period spacing between multiplets is 21.5 s for l=1 and 12.7 s for l=2. For stars like PG1159-035, these mean period spacings depend mainly on the stellar mass, with little dependence on other parameters. The observed period spacings thus allow an accurate estimate of the stellar mass; in this case the seismic mass is $0.59 \pm 0.01~M_{\odot}$ (Kawaler 1993). (5) Departures from uniform period spacings can be interpreted in terms of the chemical stratification of the WD envelope. The fairly large ($\pm 10\%$) observed variations from the mean spacing can be modeled surprisingly well using either evolutionary models starting from post-AGB progenitors, or using less restrictive structural models, as shown in Figure 5, (Kawaler 1993). The model-fitting process gives what appear to be secure values (listed in the figure) for the mass, the effective temperature, the mass of the helium-enriched surface layer, and the surface helium abundance.

IV. Pulsations in Sun-Like Stars

The best-studied and understood stellar pulsator is of course the Sun. There is no cause to believe that the Sun is an extraordinary star of its type, so it is reasonable to expect that other stars like the Sun pulsate in much the same way the Sun does. Unfortunately, the amplitudes of the solar p-modes are so small that detecting them on distant stars is a challenge: in order to justify the efforts required to do so, it is prudent to examine in considerable detail the kinds of information that one would get, should p-mode frequencies of other stars become available.

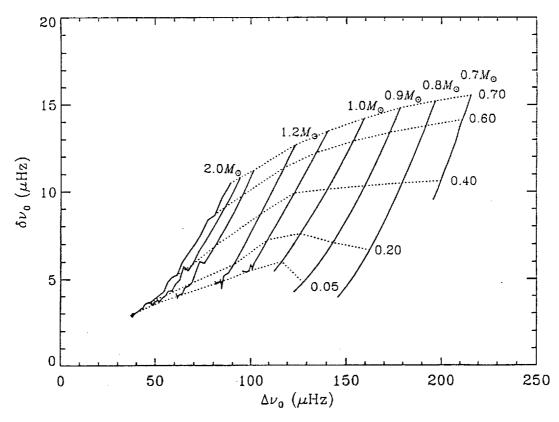


Fig. 2. The "asteroseismic H-R diagram" of Christensen-Dalsgaard (1988), showing the variation in large $(\Delta\nu_0)$ and small $(\delta\nu_0)$ frequency separation with stellar mass and age. Mass is constant along solid lines; age (parameterized by the central hydrogen abundance) is constant along dotted lines.

The first efforts to estimate the information content of oscillation frequencies for Sunlike stars were made by Ulrich (1986, 1988) and by Christensen-Dalsgaard (1988). These authors computed the sensitivity of the frequency separations $\Delta\nu_0$ and δ_{nl} to changes in the stellar mass M and age τ ; Ulrich (1986) also considered changes in the initial composition parameters Y (the helium abundance), Z (the heavy element abundance), and in the mixing length ratio α . They concluded that if the composition were known, then measurement of the two frequency separations would allow the stellar mass and age to be estimated with useful precision. These results are summarized in the so-called "asteroseismic H-R diagram," shown in Figure 2. If one assumes that individual mode frequencies may be measured with precision comparable to the mode linewidth (unknown for other stars, but typically 1 μ Hz for the Sun), then Figure 2 suggests that frequency separations could be used to determine stellar masses to within a few percent, and ages to within perhaps 5% of the main-sequence lifetime.

Brown et al. (1994) performed a more complete treatment of the problem of estimating model parameters from p-mode frequencies. Estimates of the age, mixing length, and mass of field stars can be substantially improved by the addition of oscillation frequencies. Indeed, without the frequency data, parameters such as the age are essentially unconstrained, and must be estimated from more general considerations, such as the age of the galaxy. The relative improvement in errors is greatest for distant stars, for which astrometric data are relatively unreliable. The lowest absolute errors, however, occur for nearby stars with high-quality astrometry. It develops that oscillation frequency data is usually unhelpful in constraining the heavy element abundance Z. In the best field star cases, one should be able to reach errors in mass and mixing length of about 3%, and in helium abundance and age of about 12%. Errors of this size would be interesting from the point of view of galactic evolution if they could be obtained for a good sample of stars near the Sun. They are not, however, small enough to allow tests of the physics of stellar structure theory.

A more interesting situation occurs if mode frequencies can be obtained for both stars in a well-observed visual binary (α Cen, for example). In such a case the two stars may be assumed to have the same age, distance, and initial composition, so that the number of parameters required to describe the system is less than twice that for a single star. Moreover, some new observables (the orbital data) provide fundamentally new sorts of information. One result is that parameter errors become smaller for binaries than for field stars. A more important difference is that, with many more observables than model parameters, one may search for inconsistencies between the observations and the best-fit model. If significant inconsistencies are found, then significant errors must exist in the model of the star system. In this way, it may be possible to detect errors in the physics underlying the calculation of stellar structure. For example, observed properties of a binary system (chosen to be similar to the α Cen system) were constructed using one set of opacity tables, but were fit to a model based on different (but plausible) opacities. Since the "true" and assumed models of the system employed different physics, no combination of model parameters could match the constructed observations exactly. The residuals between the "true" observations and those implied by a best fit to the model using OPAL opacities are generally large enough to be detected in spite of observational errors, and the pattern of discrepancies provides clues to the nature of the error in the assumed model. Not all modifications of the input physics result in changes that are as large as those in this example, and the degree to which different physical effects may produce similar sets of residuals is not yet known. Nonetheless, it seems reasonable that oscillation frequencies, if available, would not only allow measurement of the structural parameters of stars, but would also place constraints on at least some aspects of stellar evolution theory.

V. Expected Characteristics of the Pulsations

Having established the utility of pulsation information, one must next ask what the

observable properties of stars accessible to FRESIP might be. In what follows, I shall first describe expectations for Sun-like stars, extrapolating as needed from the Sun itself to other, similar stars. Then I shall summarize in tabular form the key observational properties of all of the multimode pulsators that might be observed by FRESIP.

Figure 3 shows the power spectrum of solar p-modes as measured with the IPHIR full-disk photometer while en route to Mars on the Soviet Phobos spacecraft (Toutain & Frölich 1992). The IPHIR instrument measured the brightness in several colors, integrated over the visible disk of the Sun, using silicon diode photometers. Except for their low noise level, these observations are thus closely analogous to normal photometric observations of stars. Figure 3 illustrates several important aspects of the solar p-modes. First, the mode frequencies are very well defined, with typical quality factors Q of several thousand. Second, the mode amplitudes are large only within a restricted frequency range, between roughly 2500 and 4000 μ Hz. Within that range, the low-degree modes to which IPHIR is sensitive are indeed almost evenly spaced in frequency, and in spite of the compressed frequency scale of this figure, many close pairs of modes (corresponding to l=0.2 or to l=1,3) may be seen. The separation between pairs of modes turns out to be roughly 68 μ Hz = $\Delta\nu_0/2$; the separation between modes making up a given l=0,2 pair is about 9 μ Hz. The amplitudes of the pulsations are quite small: the largest peaks near 3000 μ Hz have power corresponding to amplitudes $\delta l/l$ of only about 3 × 10⁻⁶.

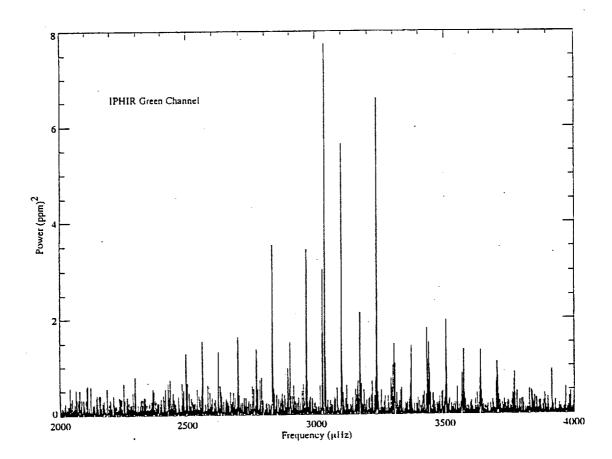


Fig. 3. Power spectrum of one month of disk-integrated solar intensity measured by the green channel of the IPHIR experiment. From data provided by C. Frölich (see Toutain & Frölich 1992).

It is fairly clear that the source of energy for the Sun's p-modes is acoustic noise generated by high-speed convective motions within a few scale heights of the solar surface (Cox et al. 1991). This implies that all stars with vigorous surface convection zones (which is to say, all stars with spectral types later than roughly F5) should support p-modes that are more or less similar to those observed in the Sun. Christensen-Dalsgaard and Frandsen (1983) estimated oscillation amplitudes as a function of Teff and surface gravity, assuming radiative damping as the dominant energy sink and a now-obsolete form of the theory of convective mode excitation. The results suggest a weak dependence of mode surface amplitude on surface gravity (cf. Eq. 9). Their results may be parameterized as

$$\frac{v_*}{v_{\odot}} = 2\left[\left(\frac{g_*}{g_{\odot}}\right)^{0.6} + \left(\frac{g_*}{g_{\odot}}\right)^{4.5}\right]^{-1} \tag{10}$$

where v_* is the typical rms velocity amplitude of the largest stellar mode, v_{\odot} is the same quantity for the Sun, and g_* and g_{\odot} are the stellar and solar surface gravities, respectively. This estimate is very uncertain, and of course says nothing about the likely lifetimes of p-modes on other stars.

A rough estimate can be made of the frequency range within which oscillations would be seen on distant stars. In the WKB approximation, the reflection of sound waves as they propagate upward through a stellar envelope is governed by the behavior of the acoustic cutoff frequency, ω_{ac} . Waves propagate upward until the local value of ω_{ac} becomes greater than the wave frequency, and then they reflect. In the simplest (isothermal layer) approximation, which is adequate for our purposes, one may write

$$\omega_{ac} = \frac{c}{2H} \propto gT^{-1/2} , \qquad (11)$$

where c is the local sound speed, H is the pressure scale height, g is the gravitational acceleration, and T is the temperature. In stellar atmospheres, ω_{ac} reaches a maximum, ω_{ac0} , in the photosphere, where the temperature is minimum. Waves with frequencies above ω_{ac0} never reflect, but rather continue propagating into the tenuous outer parts of the stellar atmosphere. As a result, p-modes with frequencies above ω_{ac0} are not expected to attain significant amplitudes. On the other hand, modes with frequencies much smaller than ω_{ac0} reflect deep in the stellar envelope. This reduces the surface amplitude for a given mode energy, and moreover reduces the coupling between the mode and the nearsurface convective driving source. These considerations suggest that maximum p-mode amplitudes should be found at frequencies that are a modest fraction (roughly 0.6, in the Sun) of ω_{ac0} . From Eq. (11), it follows that the expected frequency of maximum p-mode amplitude should scale as $gT_{\text{eff}}^{-1/2}$. If one adopts the scaling appropriate to the Sun, this implies cyclic frequencies ranging from about 1 mHz (for F-type subgiants) to about 10 mHz (for M dwarfs). The foregoing suggests that the most attractive targets for stellar oscillation searches should be stars that have lower surface gravity and are more luminous than the Sun, since such stars should have larger amplitudes (cf. Eq. 10) and should pulsate with longer periods (simplifying many observational problems).

Where stars of non-solar type are concerned, it is risky to generalize about oscillation properties because of the wide range of physical conditions involved, and because of our poor understanding of many of the important processes. Table I nevertheless attempts to do just that. For each of the categories of star that may yield asteroseismological information (including δ Scuti and roAp stars, which I have not mentioned here – see Brown & Gilliland 1994 and Kurtz 1990 for a discussion), the table lists: (1) FRESIP m_V , an estimate (guess!) of the visual magnitude of the brightest object of this class

to fall within the FRESIP field of view. The arguments leading to this guess are purely statistical; I have done no searches. (2) ν_{max} (μHz), a typical pulsation frequency. (3) $\Delta\nu_0$, and (4) $\delta\nu_0$, typical large and small frequency separations (whatever these may mean, given the context). (5) $(\delta I/I)_{rms}$, a typical photometric amplitude for the largest modes. The feasibility of observing (either from the ground or from space) pulsations such as those described in the table are discussed by Gilliland (these proceedings).

TABLE I.					
Star Type	FRESIP m_V	$ u_{max}$	Δu_0	δu_0	$(\delta I/I)_{rms}$
White Dwarf δ Scuti roAp	14 (?) 8 10	2000 200 3000	100	1 - 10	0.03 0.01 3×10^{-3}
Sun-Like	8	3000	100	10	3×10^{-6}

REFERENCES

Brown, T.M., Christensen-Dalsgaard, J., Weibel-Mihalas, B., Gilliland, R.L. 1994. Ap.J. (submitted)

Brown, T.M., Gilliland, R.L. 1994. Annu. Rev. Astron. Astrophys. 32, (in press)

Christensen-Dalsgaard, J. 1988 In Advances in Helio- and Asteroseismology, IAU Symposium No. 123, ed. J. Christensen-Dalsgaard and S. Frandsen, p. 295. Dordrecht: Reidel

Christensen-Dalsgaard, J., Frandsen, S. 1983. Solar Phys. 82: 469

Cox, A.N., Chitre, S.M., Frandsen, S., Kumar, P. 1991. In Solar Interior and Atmosphere, eds. A.N. Cox, W.C. Livingston, and M.S. Matthews, p. 618. Tucson: University of Arizona Press

Cox, J.P. 1980. Theory of Stellar Pulsation Princeton University Press, Princeton.

Däppen, W., Dziembowski, W.A. and Sienkiewicz, R. 1988. In Advances in Helio- and Asteroseismology, IAU Symposium No. 123, ed. J. Christensen-Dalsgaard and S. Frandsen, p. 233. Dordrecht: Reidel

Hansen, C.J., Cox, J.P., Van Horn, H.M. 1977. Ap.J. 217: 151

Kawaler, S.D. 1993. Ap.J. 404: 294
Kawaler, S.D., Weiss, P. 1990. In Progress of Seismology of the Sun and Stars ed. Y. Osaki and H. Shibahashi, p. 431.

Kawaler, S.D., Winget, D.E., Iben, I. Jr., Hansen, C.J. 1986. Ap.J. Lett. 306: L41

Kurtz, D.W. 1990. Annu. Rev. Astron. Astrophys. 28: 607

Tassoul, M. 1980. Ap.J. Suppl. 43: 469

Toutain, T., Frölich, C. 1992. Astron. Astrophys. 257: 287

Ulrich, R.K. 1986. Ap.J. Lett. 306: L37

Ulrich, R.K. 1988. In Advances in Helio- and Asteroseismology, IAU Symposium No. 123, ed. J. Christensen-Dalsgaard and S. Frandsen, p. 299. Dordrecht: Reidel

Winget, D.E., Hansen, C.J., Van Horn H.M. 1983. Nature 303: 781

Winget, D.E., Nather, R.E., Clemens, J.C., Provencal, J., Kleinman, S.J. et al. 1991. Āp.J. 378: 326